

Computationally-Effective Worst-Case Estimation of Currents in Transmission Lines for EMC Diagnostics of Big Systems

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Abstract—A technique for calculation of currents in transmission lines in a wide frequency range is proposed. The technique is based on a generalization of the lumped-circuit calculation methods to the case of long transmission lines by the introduction of refinement functions. A worst-case model for the amplitude-frequency characteristics of currents in the transmission line is developed on the basis of the proposed technique. The model is intended for diagnostics (express analysis) of electromagnetic compatibility in complexes of radio and electronic equipment. Worst-case nature of the model is provided by construction of the envelope of the current's maxima (observed at resonances) for the high-frequency band. The validity of the model is checked by comparison with the results of numerical simulation of single wire above ground plane, two-wire, coaxial, and triaxial transmission lines in a wide range of parameter values: the frequency is varied from 10 kHz to 8 GHz, the length of the line is from 20 cm to 5 m, height of the line above the ground surface is from 2 mm to 100 mm, the ratio of the load capacitance (inductance) to the line capacitance (inductance) is from 10^{-40} to 10^4 . Straight and piecewise-straight lines (both parallel and non-parallel to the ground plane) are considered during the validation.

Keywords—*electromagnetic compatibility, circuit analysis, RLC circuits, parameter estimation, transmission lines*

I. INTRODUCTION

As a rule, the analysis of the desired signal does not cause great difficulties in calculation of transmission lines because such analysis is usually carried out in low-frequency approximation or in approximation of well-matched line. But for the analysis of electromagnetic compatibility (EMC), it is necessary to consider the transmission of various interfering signals (such as unwanted components of radio transmitters' output spectra, noises in power supply and control circuits) which may have a very wide frequency range. For estimation of the interfering signal levels, it is reasonable to analyze the transmission lines in a wide range of frequencies and in the worst-case approximation.

To determine electromagnetic fields radiated by a transmission line (for example, by means of the technique developed in [1]), it is necessary to know currents in the line. A

technique for calculation of the currents must satisfy the following special requirements [2], [3] caused by its major application to EMC diagnostics of big systems (such as aircraft or ship). The technique must have a worst-case behavior: it must prevent underestimation of the currents for all frequencies under consideration. The technique must also provide a high computational efficiency and a stability of calculation results against errors in given parameters of the transmission line, source, and load.

All known methods do not satisfy the mentioned requirements. Circuit theory is applicable only in the low-frequency band, in which the transmission line can be represented as a circuit with lumped parameters [4]. Exact solutions obtained in theory of electrically-long transmission lines are applicable only for uniform lines (the loads at both ends of the line must also be given) [5]. Numerical solution of the transmission-line equations causes large computational burden [6]. The solution of the problem at high frequencies has a resonant behavior, thus the solution is not worst-case and not stable against errors in initial data.

The objective of this work is to develop and to validate such technique for calculation of currents in transmission lines that satisfies the special requirements presented above.

The paper is organized as follows. In Section II, parameters of the transmission line are defined, and then simplifications and approximations made for the development of the technique are introduced. In Section III, the basic relations used for current calculation are presented, and the worst-case calculation technique is developed for various frequency bands. The validation of the technique is performed in Section IV. Applications of the model, its advantages, possible generalizations, and directions for further development are discussed in Conclusion.

II. PHYSICAL MODEL OF TRANSMISSION LINE AND SIMPLIFICATIONS MADE FOR DEVELOPMENT OF TECHNIQUE

In the general case, transmission lines consist of wires, lumped and distributed sources and loads, closing stubs, shielding surfaces (one- and multilayer), dielectric coatings [4].

Transmission lines under consideration are situated directly near to conducting (metallic) surfaces (Fig. 1). The line is divided into parts, which are parallel to the surface (or surfaces). This approach provides regularity of line's parts, i.e. constancy of per-unit capacitance and inductance of the part.

Distance between the conducting surface and the axis of the transmission line is denoted by h . The length of the regular line is determined as a sum of lengths of its regular parts. For example, lengths of the lines shown in Fig. 1 are $l_1 = l_{1_1} + l_{1_2}$ and $l_2 = l_{2_1} + l_{2_2} + l_{2_3}$, respectively.

If the rectilinear part of the transmission line is situated angularly to a conducting surface, then the values of capacitance and inductance are calculated for the average height of the part over the conducting surface [2]: $h_{midl} = (h_{max} + h_{min})/2$, where h_{max} and h_{min} are the maximal and minimal heights, correspondingly.

Parameters characterizing the single unshielded wire are the diameter of central metallic conductor $d = 2r$, diameter of the wire with insulation D , permittivity of the insulator ϵ_r , and conductivity of the metal σ .

The following simplifications and restrictions are accepted for developing the technique of the line currents' calculation in the framework of the introduced physical model. The conducting surface nearest to the wire may be considered as a ground plane if its capacitance is much more than the capacitance of the transmission line. The material of the ground plane is considered as perfectly conducting material (PEC). Lossless lines are considered only. The line radiation power is assumed to be much less than the transmitted power from the source to the load along the line. To satisfy these conditions, the total resistance of the line must be 10 times less than the load resistance. Capacitance and inductance of the source and the load may be very small, but not equal to zero. The current distribution in a cross section of wires and shields is axially symmetrical. Insulators conductivity is zero. Dielectrics are homogeneous and isotropic. Permittivity of dielectrics does not depend on frequency and amplitude of electromagnetic oscillations. There are no magnetic materials in the structure of transmission lines: $\mu_r = 1$.

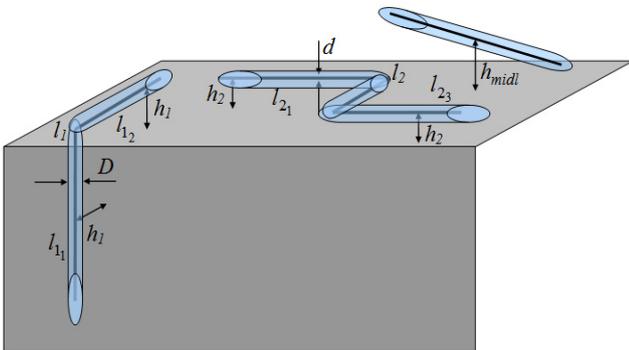


Fig. 1. The physical model geometry for single wire in dielectric insulation over conducting surface

III. TECHNIQUE FOR CALCULATION OF CURRENTS IN TRANSMISSION LINES

A. Low-Frequency Band. Simplified Equivalent Circuit of Transmission Line

As a rule [4], the low-frequency band contains frequencies f that satisfy the condition $\lambda \geq 10l$, where $\lambda = c/f$ and c is the velocity of light in free space.

A simplified model of the transmission line for the low-frequency band is represented by symmetrical circuit with lumped parameters (Fig. 2).

Connection of the source resistance R_S , inductance L_S , and capacitance C_S (and the connection of the corresponding load elements R_L , L_L , and C_L) presented in Fig. 2 is caused by physical adequacy requirement for the model [2], [3]. This connection makes it possible to simulate the properties of practical sources and loads by fitting their amplitude-frequency characteristics (AFCs) in a given frequency range.

It is empirically found that the definition of a current I_S at the source end and a current I_L at the load end of the transmission line is sufficient to calculate fields radiated by the line [1]. These currents are determined by ammeters shown in Fig. 2.

The load impedance (see Fig. 2) is defined as

$$Z_L = \frac{(R_L + j\omega L_L)(1/(j\omega C_L))}{R_L + j\omega L_L + 1/(j\omega C_L)}, \quad \omega = 2\pi f. \quad (1)$$

Impedance of the transmission line together with the load is

$$Z_{TL} = 0.5Z_T + \frac{(Z_L + 0.5Z_T)(1/(j\omega C_T))}{Z_L + 0.5Z_T + 1/(j\omega C_T)}, \quad (2)$$

$$Z_T = R_T + j\omega L_T.$$

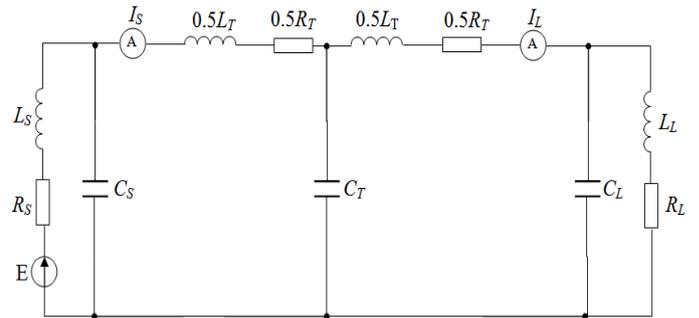


Fig. 2. Simplified symmetrical model of a transmission line. R_T is resistance of the line, L_T is inductance of the line, C_T is capacitance of the line

The total impedance of the simplified circuit model of the transmission line (ref. Fig. 2) takes the form:

$$\begin{aligned} Z_{Full} &= Z_S + \frac{(Z_{TL})(1/(j\omega C_S))}{Z_{TL} + 1/(j\omega C_S)}, \\ Z_S &= R_S + j\omega L_S. \end{aligned} \quad (3)$$

The current I_S at the source end is defined by formula:

$$I_S = E(1 - Z_S / Z_{Full}) / Z_{TL}. \quad (4)$$

For the current I_L at the load end, one can write:

$$I_L = \frac{E(1 - Z_S / Z_{Full})(1 - 0.5Z_T / Z_{TL})}{Z_L + 0.5Z_T}. \quad (5)$$

Taking into account the skin-effect, one can express the resistance R_T of a single wire as

$$R_T = l / (2\pi\sigma\delta(r - \delta(1 - \exp(-r/\delta))))), \quad \delta = (\pi\mu_0\mu_r\sigma f)^{-0.5}, \quad (6)$$

which generalizes the formula [4].

Capacitance C_T and inductance L_T of the insulator-coated wire placed above the ground plane are defined by [4]

$$C_T = \frac{2\pi\epsilon_{eff}\epsilon_0 l}{\ln(4h/d)}, \quad L_T = \frac{\mu_r\mu_0 l}{2\pi} \ln\left[\frac{4h}{d} - 1\right], \quad (7)$$

where ϵ_0 and μ_0 are SI system constants, and the effective permittivity is defined by the following expression:

$$\epsilon_{eff} = \frac{\epsilon_r \ln(4h/d)}{\ln(D/d) + \epsilon_r \ln(4h/D)}. \quad (8)$$

Existence of six resonances is possible in the circuit model presented in Fig. 2. The following resonances are the most important for our purposes. Determined by parameters of the load, the parallel resonance of current I_L arises at a frequency

$$f_L = \left(2\pi\sqrt{L_L C_L}\right)^{-1}. \quad (9)$$

The first parallel resonance of the current I_S is determined by parameters of the load and the line. The frequency of this resonance is near to

$$f_{1S} = \left(2\pi\sqrt{(L_L + 0.5L_T)(C_L + C_T)}\right)^{-1}. \quad (10)$$

For a circuit containing half of the line inductance and parallel connection of the load capacitance and the line capacitance, the frequency of series (voltage) resonance is defined by the following formula:

$$f_U = \left(2\pi\sqrt{0.5L_T(C_L + C_T)}\right)^{-1}. \quad (11)$$

For a circuit containing half of the line inductance and series connection of the load capacitance and the line capacitance, the parallel resonance arises at the frequency

$$f_{2S} = \left(2\pi\sqrt{0.5L_T C_T C_L / (C_T + C_L)}\right)^{-1} \quad (12)$$

and causes the minimum of the source current I_S amplitude.

The maximal resonance frequency which can arise in the simplified symmetrical circuit is defined by the formula:

$$f_{max} = \left(2\pi\sqrt{\min(0.5L_T, L_L, L_S) \cdot \min(C_T, C_L, C_S)}\right)^{-1}. \quad (13)$$

For description of resonances arising at low frequencies, it is necessary to take into consideration the relative errors $\delta L_L, \delta C_L$ in the load inductance and capacitance values. The half-width of the corresponding frequency range near to the resonance frequency f_r given by (9) – (12) is expressed as

$$\Delta f_r = 0.5(\delta L + \delta C)f_r. \quad (14)$$

To prevent underestimation of the currents in the frequency range $f \in [f_r - \Delta f_r, f_r + \Delta f_r]$ near to the resonance frequency, the worst-case approach is used. For parallel resonance, the worst-case estimation of the current is the maximum from amplitudes determined at the frequency range bounds. The worst-case estimation of the current for the frequency range around the series resonance is the resonance amplitude of the current.

B. Worst-Case Generalization of Low-Frequency Model to the Resonance- and High-Frequency Bands

The analysis of electrically-long transmission lines by numerical methods is carried out by dividing the line into small regular elements. To generalize the technique for analytical calculation of the currents, a coefficient $N(f)$ is introduced in the high-frequency band. This coefficient is determined by the number of elements having the length of 0.1λ and falling into the transmission line length:

$$N(f) = 2 + (lf / (0.1c)). \quad (15)$$

Transformation of the model from the symmetrical form (ref. Fig. 2) to the nonsymmetrical form (Fig. 3) with the

frequency-dependent coefficients $N(f)$ and $M(f)$ is called the dynamical generalization of the model.

It is determined empirically that the values $N(f)^{-1} = 0.15$ and $M(f)^{-1} = 0.5$ provide the worst-case estimation of the current amplitudes in the frequency range $f \in [c/10l, c/2.5l]$. Therefore, formulas (2) and (5) for calculation of the currents in that frequency range take the following form:

$$\begin{aligned} Z_{TL} &= 0.15 Z_T + \frac{(Z_L + 0.5 Z_T)(1/(j\omega C_T))}{Z_L + 0.5 Z_T + 1/(j\omega C_T)}, \\ I_L &= \frac{E(1 - Z_S/Z_{Full})(1 - 0.15 Z_T/Z_{TL})}{Z_L + 0.5 Z_T}, \end{aligned} \quad (16)$$

and formulas (3) and (4) are not changed.

It is necessary to determine the frequency of transition from the low-frequency model to the high-frequency generalization. To solve this problem, the frequency f_0 (which depends on transmission line parameters only) is introduced:

$$f_0 = (2\pi\sqrt{0.5L_T C_T})^{-1}. \quad (17)$$

Frequency f_0 (17) approaches the frequency of the series resonance f_U (11) if the load capacitance is less than the line capacitance ($C_L < C_T$). But if the load capacitance exceeds the line capacitance, then the frequency f_0 approaches the frequency f_{2S} (12) of the parallel resonance (that resonance is observed for the current I_S only). It is reasonable to choose the transition frequency near to the series resonance frequency for uniform description of both currents (I_S and I_L) and for prevention of discontinuities in AFCs of the currents (the discontinuities may arise as a result of transition between models intended for different frequency bands).

The corrected frequency is introduced as

$$f_{0n} = (2\pi\sqrt{0.15L_T C_T})^{-1}. \quad (18)$$

The corrected frequency (18) can be used as the transition frequency in case of $C_L > C_T$ because it corresponds to the series resonance frequency in the circuit redefined according to (16). For the frequency range from f_0 to f_{0n} , calculation of currents is carried out by the use of (3), (4), and (16). The transition frequency from the low-frequency band to resonance band is obtained by combination of formulas (17) and (18):

$$f_{trans} = \begin{cases} f_0, & C_L \leq C_T \\ f_{0n}, & C_L > C_T \end{cases}. \quad (19)$$

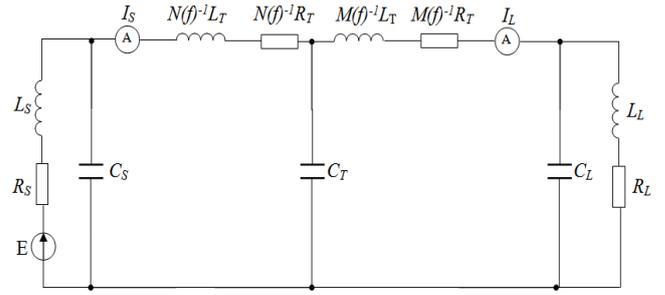


Fig.3. The generalized model of the transmission line for developing the worst-case approximation of AFCs in the high-frequency band

The resonance band for the proposed technique is the frequency range

$$f_{trans} \leq f < f_{end}, \quad f_{end} = \min(f_{max}, 4 \cdot 10^{10}) \text{ Hz}, \quad (20)$$

where f_{trans} is defined by (19) and f_{max} is calculated by (13).

The high-frequency band for the proposed technique is defined by the following condition:

$$f \geq f_{end}. \quad (21)$$

For the resonance-band frequencies (20), the current amplitude is calculated according to the formula

$$I_{S_{res}} = I_{L_{res}} = E/(R_T + R_S). \quad (22)$$

In the high-frequency band, formulas (2) – (5) underestimate the currents in comparison with the results of numerical modeling. This is caused by a large inductive reactance of the simplified symmetrical circuit in the high-frequency band. But the simplified circuit does not describe the distribution of impedance along the line length.

A simplified empirical method for calculation of the current I_S in the high-frequency band is as follows: the results given by the symmetrical lumped-circuit model are multiplied by coefficient (15). This makes it possible to obtain the worst-case estimation for the envelope of the source-current AFC maxima in the high-frequency band:

$$I_{S_{high}} = |I_S|N(f). \quad (23)$$

To determine the worst-case estimation of the current I_L , a value computed by the symmetrical lumped-circuit model should be multiplied by square of coefficient (15):

$$I_{L_{high}} = |I_L|(N(f))^2. \quad (24)$$

By combining the solutions for low-frequency, resonance, and high-frequency bands, we obtain the worst-case estimation of the current I_{SW} at the source end:

$$I_{SW} = \begin{cases} I_S & f \leq f_0 \\ I_{Sn} & f_0 < f \leq f_{trans} \\ I_{Sres} & f_{trans} < f < f_{end} \\ \min(I_{Sres}, I_{Shigh}) & f \geq f_{end} \end{cases}, \quad (25)$$

where I_S is calculated for the simplified symmetrical circuit by (4) and (2), I_{Sn} is calculated for its generalization by (4) and (16), the current I_{Sres} is defined by (22), and the current I_{Shigh} is given by (23). The choice of minimum value between the currents I_{Sres} and I_{Shigh} provides a smooth transition from the resonance band to the high-frequency band.

The worst-case estimations of the current I_{LW} at the load end can be combined similarly:

$$I_{LW} = \begin{cases} I_L & f \leq f_0 \\ I_{Ln} & f_0 < f \leq f_{trans} \\ I_{Lres} & f_{trans} \leq f < f_{end} \\ \min(I_{Lres}, I_{Lhigh}) & f \geq f_{end} \end{cases}, \quad (26)$$

where I_L is calculated by (5), I_{Ln} is computed on basis of its generalization (16), I_{Lres} is defined by (22), and I_{Lhigh} is given by (24).

The developed technique may be generalized to the cases of coaxial and triaxial lines, a straight-wire pair, or a twisted pair. To calculate per-unit-length parameters of such lines, the set of appropriate formulas [4] can be used in place of (6) and (7).

IV. VALIDATION OF TECHNIQUE INTENDED FOR WORST-CASE ESTIMATION OF CURRENTS IN TRANSMISSION LINE

The proposed technique is validated by comparison of the currents' AFCs calculated by the developed technique with the numerical computation results considered as etalons. The maximal frequency f_{et} of the etalons is limited by the applicability range of the multiconductor transmission line method ($\Delta \leq 0.1\lambda$, where Δ is the maximal dimension of transmission line in the cross-section).

The validation was carried out for two frequency ranges: from 50 kHz to 200 MHz with the frequency mesh step of 25 kHz (to determine the maxima of the current's AFCs at resonances) and from 12 MHz to f_{et} at the mesh of 1000 frequencies (for analysis of the AFCs in the high-frequency band). Examples of the currents' AFCs obtained by numerical modeling and by the developed model (25), (26) are shown in Fig. 4.

Validation was performed for the following types of transmission lines.

1. Single wire over ground plane. The following parameter values were used for the validation. The central conductor radius is 0.18, 0.50, 0.56 mm; the radius of the wire with insulation is 0.50, 1.25, 1.50 mm; the permittivity of the insulation is 1.0, 2.3, 3.0; the height of the wire over ground plane is 2, 3, 7, 10, 40 mm (if the wire is parallel to the conducting surfaces); the wire length is 0.2, 0.4, 0.6, 0.8, 1.0, 2.0 m. Wires with one or more bends (L-, S-, and U- shaped) are considered, too. Rectilinear wires located angularly to the conducting surface are also considered: the length of the wire projection on the ground plane is 200 mm, the height of the wire beginning point over the plane is 10 mm, and the wire end point height is 20, 50, 210 mm.

2. Single-shielded (coaxial) line with various grounding configurations: the shield is grounded at the source end, return current flows through the shield; the shield is single-grounded, return current flows through the ground plane, the shield is multiple-grounded. The following parameter values were used for the validation: the radius of the central conductor is 0.24, 0.47 mm; the internal radius of the shield is 0.74, 1.48 mm; permittivity of the dielectric located between the central conductor and the shield is 1.0, 2.3, 3.0; thickness of the shield is 0.26, 0.52 mm; both foiled and braided shields were considered; thickness of the external layer of dielectric (which has the permittivity of 3) is 0.5, 0.7 mm; the height of the line over the ground plane is 10, 20, 50, 100 mm; the line length is 0.25, 0.50, 1.0, 2.0, 5.0 m.

3. The model of double-shielded (triaxial) line was obtained by adding the external shield (the thickness of which is 0.33, 0.50 mm) and the dielectric layer around the external shield to the coaxial line. The parameters of the external dielectric layer are as follows: permittivity is 1.0, 2.3, 3.0; thickness is 0.5 mm. Eight grounding configurations that are usually implemented in practice are considered [3].

4. Balanced straight-wire pair and balanced twisted pair. The following parameter values were used for the validation: the radius of each conductor is 0.56, 1.00 mm; the radius of each wire in insulator is 1.0, 2.5 mm; the distance between the centers of the wires is 2.5, 3.0 mm; the height of line over the ground plane is 10, 20, 50 mm; the length of the line 0.5, 1.0, 2.0 m; number of twists per 1 meter is 10.0, 20.0, 50.0, 50.5.

5. Unbalanced twisted pair was modeled with the same parameter values as the balanced twisted pair.

For the technique validation, the following parameters of the source and load are used: R_L is 5, 50, 500 Ohm; L_L is 10^{-50} , 10^{-9} , 10^{-6} , 10^{-3} H; C_L is 10^{-50} , 10^{-12} , 10^{-9} , 10^{-6} F; R_S is 0, 5, 50 Ohm; L_S is 10^{-50} , 10^{-9} , 10^{-6} H; C_S is 10^{-50} , 10^{-12} , 10^{-9} F.

The underestimation of the currents' amplitudes by the worst-case model is not detected during the validation. However, a substantial difference between the results computed by the model and the etalons is observed in some cases in a frequency range near to the transition from the resonance band to the high-frequency band.

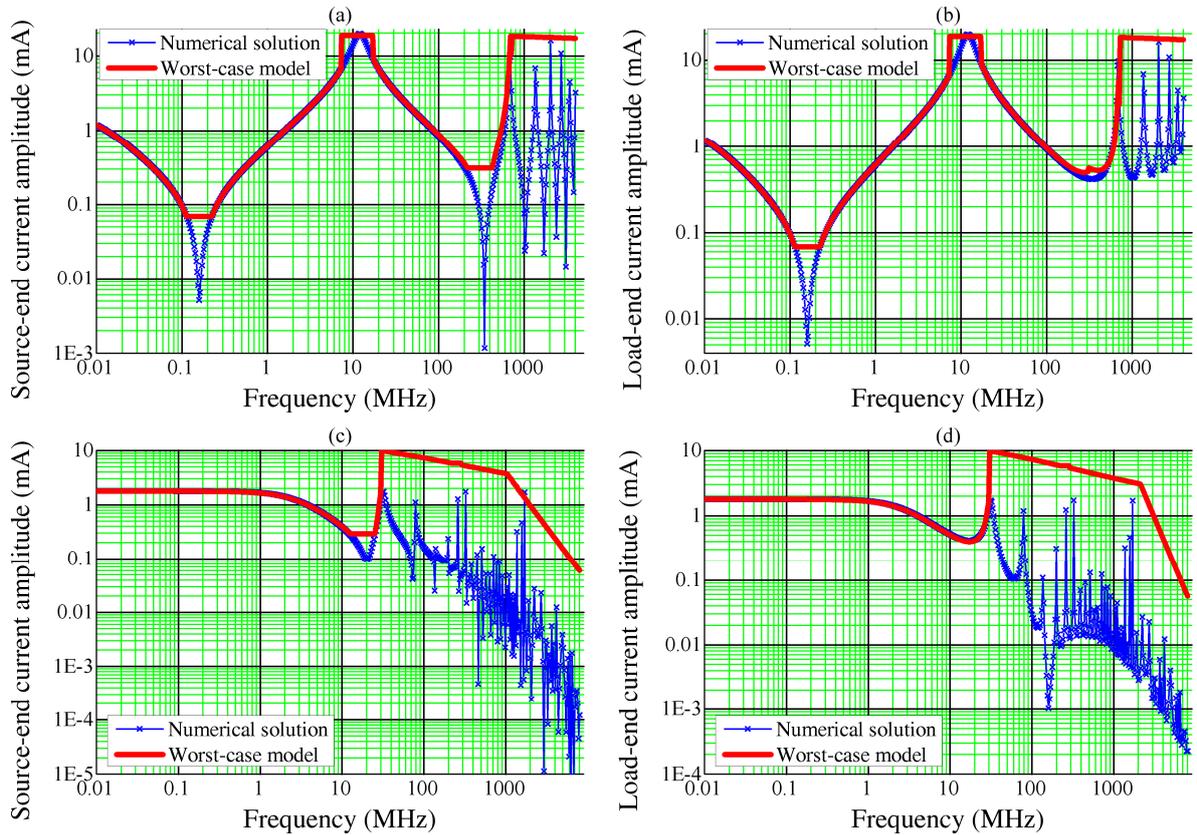


Fig. 4. Amplitude-frequency characteristics (AFCs) of currents in a single straight-line wire above the ground plane. Parameters of modeling: radius of central conductor (copper) is 0.18 mm, outer radius of its insulation is 0.50 mm. E.m.f. of source is 0.1 V. Resistances are $R_s=5$ Oh, $R_l=50$ Oh. Relative errors are $\delta R=\delta L=\delta C=0.4$. a) b) Length of line is 0.2 m, height of its axis is 7 mm, $L_s=1$ nH, $C_s=1$ pF, $L_l=1$ mH, $C_l=1$ nF; c) d) length is 2 m, height of its axis is 3 mm, $L_s=1$ μ H, $C_s=1$ pF, $L_l=1$ μ H, $C_l=1$ pF.

V. CONCLUSION

The proposed technique and the model for AFCs of the transmission line currents (25), (26), which is developed on basis of that technique, can be used for the diagnostics of EMC in big systems of radio and electronic equipment [2], [3]. The model makes it possible to calculate the fields radiated by the transmission line (for example, by means of the technique developed in [1]) and to estimate the interfering signal power dissipated in the load of the transmission line.

Physical adequacy of the model is determined by fulfillment of charge and energy conservation law in low-frequency limit (since the current calculation technique is based on Kirchhoff laws). The advantages of the technique are the wide band of frequencies under consideration, high computational efficiency, and stability to errors in initial data.

Possible generalizations and future development of the model are associated with introducing the finite conductivity of the ground plane material, accounting for dimensions and irregularities (e.g., slots) of the ground plane, and improving the models of the source and load. In order to account for

additional resonances observed near to the upper bound of the low-frequency band for coaxial line, triaxial line, and shielded twisted pair, it is reasonable to consider improved equivalent circuits describing the influence of the shields more accurately.

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