Computationally-Effective Ultra-Wideband Worst-Case Model of Electromagnetic Wave Diffraction by Aperture in Conducting Screen

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***Abstract*—A worst-case model of the plane wave diffraction by the rectangular or circular aperture is proposed. The model is intended for EMC analysis in a frequency range from 1 Hz to 40 GHz, and it is based on combining the analytic solutions of the diffraction problem for low-frequency, resonance, and high-frequency ranges. The combining is carried out by the use of matching functions which define a contribution of each analytic solution. The correctness of the proposed model was checked by comparison with numerical solutions of the diffraction problem in a wide range of the parameters' values: ratio of the wavelength to the maximal dimension (diagonal or diameter) of the aperture is from 0.05 to 300; ratio of rectangular aperture sides is from 1 to 100; ratio of the distance (from the aperture center to the observation point located behind the screen at arbitrary position) to the maximal dimension of the aperture is from 0 to 1000.**

Key words—Worst-case models, Fraunhofer diffraction, Fresnel diffraction

I. Introduction

The model of electromagnetic wave diffraction by the aperture, which is intended for EMC analysis at the level of complex systems [1], [2], should satisfy the following requirements: physical adequacy, validity in a wide range of variation of parameters, worst-case character, and stability with respect to errors in the initial data. It is also necessary to provide a high computational efficiency, which can be achieved by analytical description of the diffraction fields. The functions that describe the radiation must be continuous both in the coordinates of the observation point and in frequency.

All known analytical models are valid for certain frequency bands [3], [4], [5], but there are no clear quantitative criteria to define the boundaries of the frequency bands and spatial regions in which each analytical model can be used in practice. Solutions obtained by different analytical models produce different results on the boundaries of the frequency bands and on the boundaries of the spatial regions [6]. The speed of calculation by existing numerical methods is too low [7]. Exact solutions are sometimes characterized by irregularity [8].

The objective of this work is to develop an aperture diffraction model which satisfies the requirements given above by combining the analytical solutions for different frequency bands.

The paper is organized as follows. In Section II, we define the parameters of the problem and introduce approximations and simplifications made in the model. Section III presents the results of solving the diffraction problem for circular and rectangular apertures in different frequency bands. A wideband composite model is developed in Section IV and validated in Section V. The range of applicability of the model, its advantages and drawbacks, possible generalizations, and directions for further development are discussed in Conclusion.

II. Physical Model of Electromagnetic Wave Diffraction by Aperture

The geometry corresponding to the physical model of diffraction by aperture is shown in Fig. 1.

The screen with aperture is situated in a homogeneous isotropic dielectric. Screen is infinite in the plane *XOY*. The thickness of the screen is h (defined along *OZ* axis, $h << \lambda$). The conductivity of the screen's material is infinite (PEC).

The sizes of rectangular aperture in plane of the screen are the length a defined along *OX* axis and the width b defined along *OY* axis. For description of the circular aperture, the radius R is introduced. The characteristic size is $L_R = \max\{a, b\}$ for the rectangular aperture and $L_C = 2R$ for the circular aperture.

The incident plane wave is linearly polarized. A wave vector $\vec{k}_0$ determines a direction of propagation of the incident wave ($k_0 \equiv \left|\vec{k}_0\right| = 2\pi / \lambda_0$, λ_0 is a wavelength). Electric vector of incident wave is $\vec{E}_0$ and magnetic vector is $\vec{H}_0$. Amplitudes of the vectors are $\left|\vec{E}_0\right| \equiv E_0$ and $\left|\vec{H}_0\right| \equiv H_0$, respectively. The relationship between these vectors is expressed as

$$E_0 = Z_C H_0 \, , \; Z_C = \sqrt{(\mu\mu_0)/(\varepsilon\varepsilon_0)} \, , \tag{1}$$

where Z_C is the wave impedance, ε is the dielectric constant and μ is the magnetic permeability of environment, (ε_0 and μ_0 are SI system constants).

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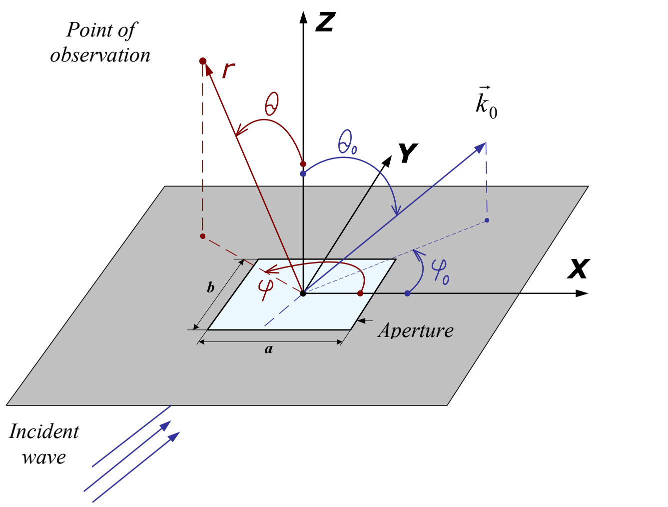

Figure 1. The physical model of diffraction of the plane electromagnetic wave by the rectangular aperture in the PEC screen.

The incidence plane is a plane determined by *OZ* axis and by vector $\vec{k}_0$. The line corresponding to intersection of the incidence plane and *XOY* plane is denoted by blue dash line in Fig. 1. The angle between this line and *OX* axis is φ_0. The angle between the wave vector and the positive normal to the screen (aperture) is θ_0.

The polarization of the incident wave is determined as follows. If the electric vector belongs to the plane of incidence, the wave is called TM-wave. If the magnetic vector belongs to incidence plane, the wave is called TE-wave. According to the problem statement, the incidence plane is specified but the polarization plane is not defined, therefore, it is necessary to consider both TM-wave and TE-wave.

The point of observation is placed behind the screen at the distance r from the center of the aperture. The polar angle of the observation point is denoted by letter φ, and the zenith angle (relative to *OZ* axis) is denoted by θ (see Fig. 1).

The problem is to determine the magnitudes of the electric and magnetic vectors (E and H , correspondingly) in arbitrary point of observation.

III. Approaches to Solving of Diffraction Problem in Different Frequency Bands

Huygens-Fresnel principle is the basis of solving of the diffraction problem. According to this principle, every point of the wave front is a source of secondary waves. The secondary waves are coherent. The new wave front is the result of interference of secondary waves. To develop an ultra-wideband diffraction model it is necessary to determine the characteristics of secondary sources and the conditions of interference.

A. Electrically-Small Aperture: Dipole Approximation

If the wavelength of incident wave is more than size of aperture, the phase difference of secondary waves is equal to zero for all point of observation. All sources of secondary waves, which belong to the area of aperture, radiate in the same phase independently of the angle of incidence. This postulate is the basis of quasi-static approximation for solving of diffraction problem in the case of electrically-small aperture or low-frequency band.

The surface currents and the electrical charges distribution are the sources of secondary waves. The currents in region nearest to the aperture influence on field's distribution in a great.

It is known, [3] that the radiation of small aperture is equivalent to the radiation of magnetic dipole.

The radiation of magnetic dipole with the axis directed along *OZ* is described by formulas [6]:

$$\begin{pmatrix} H_r \\ H_\theta \\ E_\varphi \end{pmatrix} = \begin{pmatrix} -4P_{L\,C} H_0 \cos(\theta)\left(2(k\,r)^{-2} - 2\,j(k\,r)^{-1}\right) \\ -4P_{LC} H_0 \sin(\theta)\left((k\,r)^{-2} - j(k\,r)^{-1} - 1\right) \\ 4P_{L\,C} Z_C H_0 \sin(\theta)\left(j(k\,r)^{-1} + 1\right) \end{pmatrix}, \qquad (2)$$

where $P_{L\,C} = \left(R^3 k^2 \exp(-j\,\vec{k}\vec{r})\right)/(3\pi r)$, $j^2 = -1$.

The axis of dipole must be perpendicular to the plane of polarization of incident wave. To describe the diffraction of wave polarized in *XOZ* plane by formulas (2) in our case (see Fig. 1) it is necessary to carry out the following transformation of coordinates $(x, y, z) \to (x, -z, y)$. For the wave polarized in *YOZ* plane, the transformation of coordinates takes the form $(x, y, z) \to (-z, y, x)$.

The description based on formulas (2) is correct in the case of normal incident wave and for distances more than three characteristic size of aperture $(r > 3\,L_{C,R})$.

An introduction of the angle of incidence in Dipole approximation is based on Bethe approach [3]. For electric vector, one can write:

$$\vec{E}_1 = P_{LC}\vec{k}_n \times (2\vec{H}_0 Z_C + \vec{E}_0 \times \vec{k}_n), \quad \vec{k}_n \equiv \vec{k}/|k|, \qquad (3)$$

where $\vec{E}_{0\,TM} = (E_0 \cos\theta_0, 0, -E_0 \sin\theta_0)$, $\vec{E}_{0\,TE} = (0, E_0, 0)$, and magnetic vector is defined in the usual way for TM –and TE-waves respectively. Expression (3) is true in the wave zone only $(r >> \lambda >> R)$.

Magnetic vector in the wave zone is defined for all frequency bands by expression

$$\vec{H}_1 = \left(\vec{k}_n \times \vec{E}_1\right)/Z_C\,. \qquad (4)$$

It is established, that in low-frequency band for TM-wave the magnitude of electric vector is increased with the increasing of the angle of incidence in the near-field zone ($r < 3R$). The reason of it is in increasing of the normal component of electric vector. The spatial distribution of the magnitude of electric vector changes in following way with increasing of the frequency. The forward peak of electric vector in the near-field zone is reduced, but the region of increased magnitude expands. For description of the electric field in the near-field zone for off-normal incidence, the new term is introduced:

$$\Delta E(\theta_0) = \frac{E_0 \sin\theta_0}{2\sigma(k,R)} \exp\left(-r^2 / 2(R\sigma(k,R))^2\right), \quad (5)$$

where $\sigma(k,R) = \exp(kR)$ is the parameter that determines of the change in the function (5) with frequency.

Finally, for the circular aperture (with condition $kR < 1$), the magnitude of electric vector of TM-wave may be written in the form

$$E_{L\,CTM} = \left|E_\varphi(x,-z,y)\right| + \left|E_1\right| \sin\theta_0 + \Delta E(\theta_0), \quad (6)$$

where $\left|E_\varphi\right|$ is the magnitude of electric vector in the dipole approximation (2), $\left|E_1\right|$ is defined by components of vector (3) and $\Delta E(\theta_0)$ is calculated according to expression (5).

For TE-wave solution takes the form:

$$E_{L\,C\,TE} = \left|E_\varphi(-z,y,x)\right|. \quad (7)$$

The worst-case solution for the circular aperture in low-frequency band based on the choice of maximal value corresponding to different states of polarization

$$E_{LCW} = \max\{E_{L\,CTM}, E_{L\,CTE}\}. \quad (8)$$

The magnitude of magnetic vector in the worst-case model is defined by the formula:

$$H_{LCW} = \max\{|H|, F(\theta_0) \max\{|H_{1\,TM}|, |H_{1\,TE}|\}\}, \quad (9)$$

where $|H|$ is defined by components H_r, H_θ (2) and determines the magnitude of magnetic vector in the near-field zone and intermediate zone ($r < 10L_C$), $|H_{1\,TM}|$ and $|H_{1\,TE}|$ are calculated by the formula (4) for TM-wave and TE-wave respectively. Factor $F(\theta_0)$ is introduced to provide of the worst-case character of model in the wave zone. This coefficient depends on the angle of incidence. The equation for $F(\theta_0)$ was obtained by comparison of numerical results with the results of analytical model. It takes the form: $F(\theta_0) = 2 + |\sin\theta_0|$.

Now consider the aperture in the shape of an infinitely long, narrow slot with the width a ($a < \lambda/4$). The slot is parallel to OY axis. Electric vector of incident wave $\vec{E}_0$ is directed alone OX axis. In this case, two infinitely long wires with a fictive magnetic current can be considered as sources of electric field. Distance between wires is equal to a. The spatial distribution of electric field has a cylindrical symmetry. The symmetry axis is OY axis. In the near-field zone and intermediate zone ($r < 10a$), magnitude of electric vector is determined by equations:

$$\begin{aligned}
&E_{xS} = 2K_S z\left(1/\left(A_1^2 + z^2\right) + 1/\left(A_2^2 + z^2\right)\right),\\
&E_{zS} = K_S\left(A_1/\left(A_1^2 + z^2\right) + A_2/\left(A_2^2 + z^2\right)\right),\\
&K_S = E_0 k a^2/(2\pi), \quad A_1 \equiv A_1(x) = x + a/2,\\
&A_2 \equiv A_2(x) = x - a/2.
\end{aligned} \quad (10)$$

Solution in form (10) provides the fulfillment of boundary conditions on the surface of conducting screen.

If the incident wave is polarized along the edge of narrow slot, the magnitude of electric vector on the shadow side of screen go to zero. The case of normal to the edge of the slot polarization corresponds to the worst-case model.

The electric vector for rectangular aperture with parameters a and b is defined by components:

$$\begin{aligned}
&E_{xR\,TM} = K_R z\left(\frac{D_1}{A_1^2 + z^2} + \frac{D_2}{A_2^2 + z^2}\right),\\
&E_{zR\,TM} = K_R\left(\frac{A_1 D_1}{A_1^2 + z^2} + \frac{A_2 D_2}{A_2^2 + z^2}\right),\\
&D_{1,2} = \frac{B_1}{\sqrt{B_1^2 + A_{1,2}^2 + z^2}} - \frac{B_2}{\sqrt{B_2^2 + A_{1,2}^2 + z^2}},
\end{aligned} \quad (11)$$

where $B_1 = B_1(y) = y + b/2$, $B_2 = B_2(y) = y - b/2$. The value of coefficient $K_R = E_0 k\,ab/\pi$ satisfy the requirements of the worst-case model.

The worst-case solution for the rectangular aperture corresponds to the case if $\vec{E}_0$ is normal to the longest side of the aperture. According to this condition, solution for the rectangular aperture takes the form:

$$\begin{aligned}
E_{L\,R\,W} = E_{R\,TM} + \max\left(|E_{1\,TM}|, |E_{1\,TE}|\right)\, F(\theta_0) +\\
+ \Delta E(\theta_0),
\end{aligned} \quad (12)$$

where the function of rapid decrease for the near-field zone and intermediate zone $E_{R\,TM}$ is defined by components (11). The term $|E_1|$ is defined by the components (3). The corresponding coefficient in (3) for rectangular aperture is equal to $P_{L\,R} = \left(\min(a,b)\,L_R^2 k^2 \exp(-j\vec{k}\vec{r})\right)/(3\pi r)$. The function

$\Delta E(\theta_0)$ is determined by the formula (5). In the expression (5) the parameter $\min(a,b)$ in the place of R is used.

By comparison of the results of numerical calculations and results obtained in analytical approach it is established that the rectangular apertures with the ratio of sides less then 3 may be described in the frame of circular aperture model (8), (9). In the case of ratio of sides more then 3, formula (12) provides the more adequate description.

The magnitude of magnetic vector is determined by the formula (9). The solution for the magnetic vector in the near-field zone is subjected in formula (9) too.

B. Resonance Aperture: Waveguide Approximation

If the wavelength is reduced to the value less than $\lambda = 4R$, the quasi-static approximation is not valid. In this case, the result of interference of the secondary waves in the point of observation depends on the phase difference essentially. It is a characteristic feature of diffraction in the resonance and high-frequency bands. There are a difficulties associated with the state of a wave surface in the resonance aperture. For the wavelengths of the same order as the characteristic size of the aperture, the state of wave in aperture must be considered as perturbed.

The solution of diffraction problem for the resonance aperture in the frame of worst-case model has the property typical to low-frequency and to high-frequency bands. One of possible approaches is to represent of the solution as a sum of solutions obtained for low-frequency and high-frequency bands separately.

The solution for the near-field zone is based on simplifying assumption that the state of wave surface in aperture corresponds to fundamental mode of waveguide [9]. The solution for circular aperture in resonance band may be obtained by substitution of fundamental mode $TE_{(11)}$ of circular waveguide for incident wave in generalization of Kirchhoff integral [6]. The result for the near-field zone in case of the circular aperture takes the form:

$$E_{R\,CTE} = P_{R\,C}\left(2J_0(\eta_{11}) + \eta_{11}J_1(\eta_{11}) - 2\right)G_{TE}(\theta,\varphi),$$
$$P_{RC} = Z_C H_0 \frac{2\pi^2 R^3}{\lambda^2 \eta_{11}^3}\frac{\exp(-j\vec{k}\vec{r})}{r}, \qquad (13)$$
$$G_{TE}(\theta,\varphi) = \sqrt{\cos^2(\theta) + \left(\sin(\theta)\sin(\varphi)\right)^2},$$

where J_1 is first-order Bessel function of the first kind, $\eta_{11} = 1.8412$.

Suppose that the state of incident wave in the rectangular aperture corresponds to fundamental mode $TE_{(01)}$ of rectangular waveguide one can write:

$$E_{R\,R\,TE} = P_{R\,R}\, G_{TE}(\theta,\varphi),$$
$$P_{RR} = Z_C H_0 \frac{L^3}{\lambda^2}\frac{\exp(-j\vec{k}\vec{r})}{r}. \qquad (14)$$

To develop the worst-case model the auxiliary function $G_{TM}(\theta,\varphi) = \sqrt{\cos^2(\theta) + \left(\sin(\theta)\cos(\varphi)\right)^2}$ is introduced. Since the plane of polarization does not specified, the maximal value of the angular function is defined by the following way:

$$G_W\ (\theta,\varphi) = \max\{G_{TE}(\theta,\varphi), G_{TM}(\theta,\varphi)\}. \qquad (15)$$

The magnitude of electric vector is obtained by substitution of function (15) for function $G_{TE}(\theta,\varphi)$ in equation (13). The worst-case solution for the circular aperture in resonance band takes the form:

$$E_{R\,C\,W} = \left|E_{RC\,TW}\right|\exp(-(r/2R)^2) + \left|E_{H\,C\,env}\right|, \qquad (16)$$

where $E_{H\,C\,env}$ is the worst-case generalization of function describing of diffraction by circular aperture in the wave zone for high-frequency band. This function will be defined in the next Section. The factor $\exp(-(r/2R)^2)$ limits the region of validity of the near-field zone's solution (13).

The diffraction by rectangular aperture in resonance band according to (14) and (15) is described by formula:

$$E_{RRW} = \left|E_{RR\,TW}\right|\exp(-(r/2L_R)^2) + \left|E_{H\,R\,env}\right|, \qquad 17)$$

where $E_{H\,R\,env}$ is the worst-case generalization of function describing of diffraction by rectangular aperture in the wave zone for high-frequency band (see the next Section).

For definition of magnetic vector in resonance band and in high-frequency band, the formula (4) is used.

C. Electrically-Large Aperture

The field of the incident wave in high-frequency band is considered as unperturbed.

To calculate of electric vector in the wave zone the theory of Fraunhofer diffraction is used. The magnitude of electric vector in the case of the circular aperture is defined by the equations [5]:

$$E_{H\,C} = \left|\, P_{H\,C}(\theta,r)\ J_{1C}[\chi(\varphi,\theta)]\right|$$
$$P_{H\,C}(\theta,r) = \frac{\pi E_0\, k\ R^2 \exp(-j\vec{k}\vec{r})}{r}(\cos\theta_0 + \cos\theta), \quad (18)$$
$$J_{1C}(x) \equiv 2\ J_1(\chi(\varphi,\theta))/\chi(\varphi,\theta),$$

$$\chi(\varphi,\theta) = kR\sqrt{[\sin\theta_0 - \sin\theta\cos(\varphi-\varphi_0)]^2 + [\sin\theta\sin(\varphi-\varphi_0)]^2}\ .$$

To provide of the guaranteed oversized estimate it is necessary to curry out of a smoothing of the spatial distribution of the fields. The smoothing function must pass over the points of local fields maximum.

The dependence of (18) on the angle may be smoothed by using of the worst-case approximation:

$$J_{1C}(x) \approx J_{1C}^{*}(x) \equiv \begin{cases} 1, & |x|<1.44; \\ |x|^{-3/2}\sqrt{\dfrac{8}{\pi}}\left[1+(0.62/x)^2\right], & |x|\geq 1.44. \end{cases} \quad (19)$$

The solution for the circular aperture in the far-field zone is obtained by inserting of the smoothing function (19) in the formulas (18). The result must be limited by the value $E_{\max}$. This value is defined by using of Fresnel diffraction theory. The solution for the high-frequency band takes the form:

$$E_{HCW} = \min(E_{\max}, E_{HCenv}), \quad E_{\max} = 2E_0, \\ E_{HCenv} = \left| P_{HC}(\theta, r)\; J_{1C}^{*}[\chi(\varphi,\theta)] \right|. \quad (20)$$

The approach based on Fraunhofer diffraction for the rectangular aperture transforms to the equations [5]:

$$E_{HR} = \left| P_{HR}(\theta, r)\ \mathrm{sinc}[\chi_x(\varphi,\theta)]\mathrm{sinc}[\chi_y(\varphi,\theta)], \right| \\ P_{HR}(\theta, r) = \frac{E_0\, k\, a\, b \exp(-j\,\vec{k}\vec{r})}{4\pi\, r}(\cos\theta_0 + \cos\theta), \\ \mathrm{sinc}(x) \equiv \sin x / x, \\ \chi_x(\varphi,\theta) = k\, a\ (\sin\theta \cdot \cos\varphi - \sin\theta_0 \cdot \cos\varphi_0)/2, \\ \chi_y(\varphi,\theta) = k\, b\ (\sin\theta \cdot \sin\varphi - \sin\theta_0 \cdot \sin\varphi_0)/2. \quad (21)$$

The structure of angular dependence of the solution in far-field zone for the rectangular aperture is defined by the function $\mathrm{sinc}(x)$, which has simple smoothing approximation

$$\mathrm{sinc}(x) \approx \mathrm{sinc}^{*}(x) \equiv \begin{cases} 1, & |x| \leq 1; \\ 1/|x|, & |x| > 1. \end{cases} \quad (22)$$

Then the worst-case model of diffraction by the rectangular aperture for high-frequency band takes the form:

$$E_{HRW} = \min(E_{\max}, E_{HRenv}), \quad E_{\max} = 2E_0, \\ E_{HRenv} = \left| P_{HR}(\theta, r)\ \mathrm{sinc}^{*}[\chi_x(\varphi,\theta)]\mathrm{sinc}^{*}[\chi_y(\varphi,\theta)]] \right|, \quad (23)$$

according to the approximation (22).

The choice of the limiting factor $E_{\max}$ in equations (20) and (23) according to Fresnel diffraction provide of validity of the model in the intermediate zone and in the near-field zone.

IV. Ultra-Wideband Composite Model

The solutions corresponding to the different frequency bands (ref. Section III) do not appear in a pure manner. For example, the contribution of the resonance-band solution increases continuously with approaching of the incident wave frequency to the resonance frequency. With increasing the frequency above the resonance one, the contribution of the high-frequency-band solution increases and the contribution of the low-frequency-band solution decreases. That is why the particular solutions (8), (16), and (20) are combined by summation according to the contribution of each solution, i.e., by the use of the weighted average

$$E_{CW} = \frac{E_{LCW}W_{LC} + E_{RCW}W_{RC} + E_{HCW}W_{HC}}{W_{LC} + W_{RC} + W_{HC}}, \quad (24)$$

where W_{LC}, W_{RC}, and W_{HC} are the weighting functions defined by the critical wavelength $\lambda_{cr}^{11}(R)$ for $TE_{(11)}$ mode of the waveguide associated with the circular aperture [9]:

$$W_{LC} = \exp\left(-(\lambda_{cr}^{11}(R)/\lambda)^2\right), \\ W_{RC} = \exp\left(-((\lambda - \lambda_{cr}^{11}(R))/0.5\lambda)^2\right), \\ W_{HC} = \exp\left(-(1.5\lambda/\lambda_{cr}^{11}(R))^2\right). \quad (25)$$

By combining of the solutions (12), (17), and (23) for the rectangular aperture, we obtain the formula, which is similar to (24), with the following weighting functions:

$$W_{LR} = \exp\left(-(\lambda_{cr}^{01}(L)/\lambda)^2\right), \\ W_{RR} = \exp\left(-((\lambda - \lambda_{cr}^{01}(L))/0.5\lambda)^2\right), \\ W_{HR} = \exp\left(-(1.5\lambda/\lambda_{cr}^{01}(L))^2\right). \quad (26)$$

where $\lambda_{cr}^{01}(L)$ is the critical wavelength for $TE_{(01)}$ mode of the rectangular waveguide of size $L = \min(a,b)$.

V. Verification of Developed Model

The composite model is verified by comparison of the results based on the model with the results of the numerical computation. The magnitudes of electric and magnetic fields obtained by the numerical computation are considered as standards (etalons).

To perform the verification, we developed 90 test examples for the circular aperture and 150 test examples for the rectangular aperture. In each example, the spatial distributions of E- and H-field magnitudes are computed in the volume of dimensions more than 20 times exceeding the characteristic size of the aperture in all directions. For the low-frequency band, each of the volume dimensions is not less than 5λ. Mesh spacing is equal to λ/20. Cases of TM-wave and TE-wave polarizations with the angles of incidence 0°, 30°, 60° are considered. The model was verified at the frequencies of 0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 5.0, 10.0, 30.0 GHz for circular aperture of radius *R*=5 cm and at the frequency of 30.0 GHz for circular aperture of radius *R*=10 cm. The verification was carried out at the frequencies of 0.1, 0.5, 1.0, 2.0, 5.0 GHz for rectangular apertures of sizes 1×100 1×40, 2×20, 4×10, 5×5 cm.

First, a functional model of diffraction by the circular aperture for the electric field is verified. The functional model takes into account the polarization of the incident wave, and it does not smooth the diffraction maxima and minima by (19). The difference between the field magnitude value computed by the model and the etalon value does not exceed 9 dB in 53 test examples. In 14 test examples, the differences up to 14 dB are detected in the near-field zone for the resonance frequency band only. The differences up to 18 dB are found in the rest 23 test examples in intermediate zone for the high-frequency boundary of the resonance band. The results of this verification confirm the ability to use the functional model as a basis for the development of the worst-case model.

Then, the worst-case models of diffraction by circular and rectangular apertures for the electric and magnetic fields are verified. The underestimation of the field amplitude by the worst-case models is not detected during the verification. However, in some examples corresponding to the TE-wave, a substantial difference between the results of computation by the model and the etalon is detected for the circular aperture if the angle of incidence is equal to 60^o (Fig. 2). This difference is caused by the invariance of the worst-case model under transformation of the incident wave polarization. For the rectangular aperture, the substantial difference is observed if the incident wave is polarized along the longest side of the aperture.

VI. Conclusion

The developed model can be used for calculations of EMC in local complexes of radio and electronic equipment [1], [2].

The model has the following advantages: validity in a wide frequency band, high computational efficiency, and stability with respect to errors in the initial data. Restrictions of the model result from the postulates introduced in Section II: the incident wave is plain; the screen is thin and perfectly conductive; if there is a set of apertures, the distance between their centers must more than 5 times exceed the wavelength.

Possible generalizations and future development of the model are associated with introducing the finite conductivity of the screen material and considering the interferential effects of multiple apertures. It is also reasonable to involve a model based on geometrical optics, which will significantly improve the computational efficiency for the case of high frequencies.

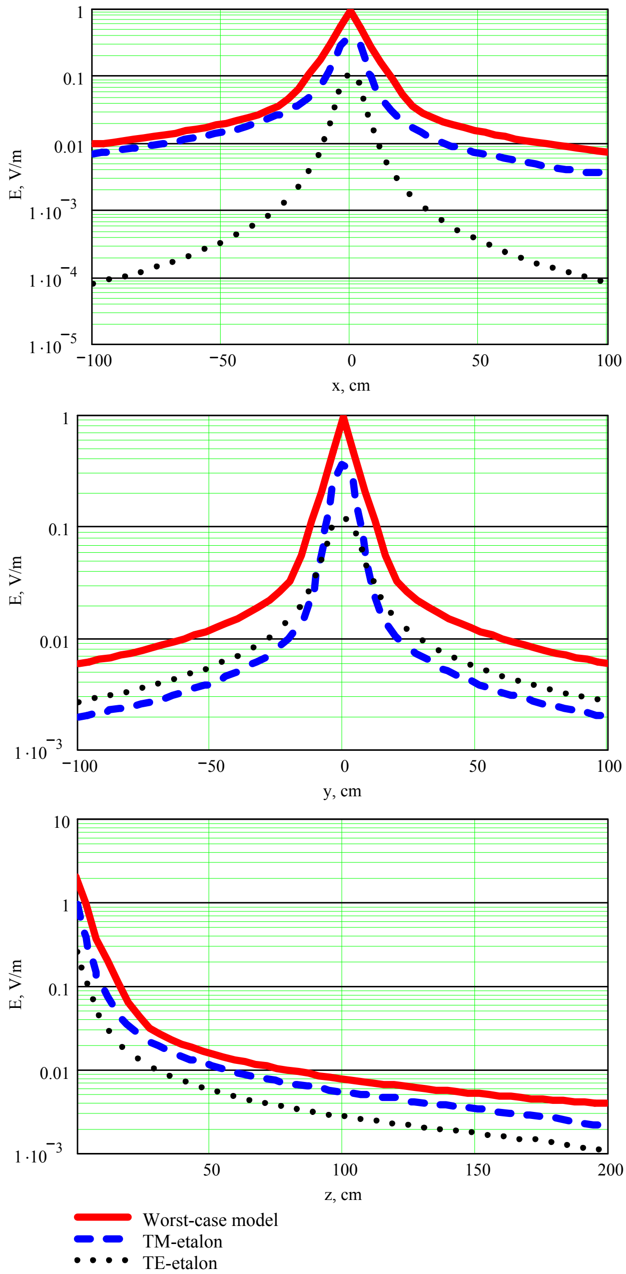

Figure 2. Comparsion of the worst-case model results to etalon. Parameters of modeling: the circular aperture with radius of 5 cm, the incident wave amplitude is 1 V/m and the frequency is 0.5 GHz, angle of incidence is 60^o, observation points are located on the lines going in parallel to coordinate axes and passing through the point with coordinates (0, 0, 3) cm.

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