Mathematical Models for Radiosignals Dynamic Range Prediction In Space-Scattered Mobile Radiocommunication Networks

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Abstract

*Paper presents mathematical models and technique for evaluating probability of strong input signals during reception in space-scattered radio communication networks, particularly high-capacity cellular networks. To achieve this aim, the **m**-dimensional Poisson system of random spatial points and a method for "differential" statistical modeling of radio environment is used. This method implies prediction of statistical characteristics of the ensemble of field (signal) strength values at the reception point. Practical implementation of these models for cellular networks system analysis and simulation is discussed.*

1. Introduction

Intensive development of mobile radio communication systems made modern urban radio environment essentially more complex. Rational frequency planning of these systems as well as shrinking of cell sizes along with the simultaneous decrease in base and mobile station transmitter powers to 1-2 watts/channel or less makes it possible to substantially lower the probability of mutual interference on the main, adjacent and spurious receive paths. Under these conditions, however, we have to consider the probability that there are comparatively strong signals from the nearest mobile stations in the location of a mobile or base station:

signals whose frequencies do not coincide with the receiving frequency but may deteriorate the communication quality due to nonlinear effects in the receiver: intermodulation, blocking, cross modulation or nonlinear local oscillator noise conversion;

signals whose frequency coincides with the receiving frequency of the weak signal from a distant mobile station (for example, if CDMA or TDMA standards are employed), which can cause the radio amplification/conversion path to be overloaded and requires for provision of a wide power adjustment range in an individual radio channel when communication with the nearest mobile stations is taking place (where this procedure is provided at the system level).

Therefore it is of practical interest to develop mathematical models and the technique for evaluating probability of signals dynamic range (or strong input signals) during reception in space-scattered mobile radio communication networks

2. Initial Models and Definitions

A) As the radio environment parameter which characterizes the strong signals at the receiver input the value of the dynamic range $\boldsymbol{D}$ of the ensemble $\boldsymbol{N}$ of input signals is used:

$$\boldsymbol{D_P} = \frac{\boldsymbol{P}_{\max}}{\boldsymbol{P_0}} = \frac{\boldsymbol{\Pi}_{\max}}{\boldsymbol{\Pi}_{\min}} = \left(\frac{\boldsymbol{E}_{\max}}{\boldsymbol{E}_{\min}}\right)^2 = \boldsymbol{D}_E^2\,, \qquad (2.1)$$

$$\boldsymbol{D} = \mathbf{10\,lg}\,\boldsymbol{D_P} = \mathbf{20\,lg}\,\boldsymbol{D_E}\,.$$

This expression contains the following terms:

power characteristics of the predominant input signal - the power flux density $\boldsymbol{\Pi}_{\max}$ and the field strength $\boldsymbol{E}_{\max}$ of the predominant field at the point of receiving antenna location; the power $\boldsymbol{P}_{\max}$ of this signal at the receiving antenna output;

the values $\boldsymbol{\Pi}_{\min}$, $\boldsymbol{E}_{\min}$ of the receiver sensitivity limit "over the field" and the receiver antenna input sensitivity value $\boldsymbol{P_0}$ of the in power measurement units.

B) As the radiowave propagation model we use the well-known hyperbolical approximation of the electromagnetic field power flux density $\boldsymbol{\Pi}$ on the distance $\boldsymbol{R}$ to its emitter:

$$\Pi = C_\nu P_{etr} \big/ R^\nu\,,\ \ P_{etr} = G_a P_{tr}\,,\ C_\nu = const, \qquad (2.2)$$

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where P_{etr} - the equivalent isotropic radiated power (EIRP), P_{tr} - the power fed to the emitter antenna; G_a - the antenna gain, C_ν - the constant (ν=2 for free space propagation, ν=4 may be used in some cases (for radiowave propagation with interference of direct and reflected rays in the far zone for the VHF range and the lower part of the UHF as well as in cases when propagation path shadowing by urban buildings, structures and foliage is initially taken into account [**1,2**], ν=2÷12 when the "regressive" in-building propagation model [**3,4**] for the UHF range is used).

C) As the receiver parameter which characterizes the receiver susceptibility to influence of strong signals outside the receiver (operating) bandwidth the value of the interference effects free input dynamic range of receiver is used [**5**]:

$$D_{in} = P_{\max}/P_0 \in \{D_{im}, D_{ds}, D_{cm}, D_{onm}, D_{inm}, D_b\}, \quad (2.3)$$

where P_0 is the lower signal power limit of a receiver antenna input sensitivity; P_{max} may be defined using, particularly, the intermodulation criterium for determining the intermodulation free dynamic range D_{im} of the receiver, the desensitization criterium for determining the desensitization free dynamic range D_{ds} of the receiver, the cross modulation criterium for determining the cross modulation free dynamic range D_{cm} of the receiver, and the respective criteria for determining the local oscillator noise mixering free dynamic range D_{onm}, the intermediate frequency paths interference free dynamic range D_{inm}, or the border frequency paths interference free dynamic range D_b of the receiver.

D) The area of the radius $R_{max}=(C_\nu P_{etr}/\Pi_{min})^{1/\nu}$ around the location point of the victim receiver with the sensitivity limit Π_{min} shall be considered as the spatial area of potential interfering interaction of radiotransmitters. Spatial arrangement of emitters within this area shall be characterized by the average spatial emitter density function of coordinates: the function $\rho(\alpha,\theta,R)$ in spherical coordinates for three-dimensional emitters arrangement or $\rho(\alpha,R)$ in polar coordinates for two-dimensional emitters arrangement; here α, θ are the azimuth and elevation angles of the signal arrival direction; R is the distance from the center of the area. We shall consider the functions $\rho(\alpha,R)$ and $\rho(\alpha,\theta,R)$ as slowly varying functions; thus the spatial density of emitters in the vicinity of the receiver location point which corresponds to the zero of coordinate system may be assumed as constant. In the general case, we shall assume $\rho=const$ for the m-dimensional vicinity of the receiver.

E) As the model of probabilistic character of spatial distribution of emitters in the vicinity of he radio receiver we shall use the known Poisson model of random distribution of points in space:

$$p_k(N_{\Delta V}) = \frac{N_{\Delta V}^k}{k!}\exp(-N_{\Delta V}), \quad (2.4)$$

where $p_k(N_{\Delta V})$ is the probability that exactly k point emitters will fall into a certain element of space ΔV if the average number of emitters within this element is equal to $N_{\Delta V}$.

F) At the initial stage of analysis we shall assume that all emitters are isotropic and have equal EIRP ($C_\nu P_{etr}=const.$) and that the receiver antenna is omnidirectional and its equivalent area is S_e ($P_0=S_e\Pi_{min}=const$).

G) We shall neglect the dependence of characteristics of the emitter antennas, receiver antenna and propagation model (2.2) characteristics on frequency assuming that the transmitting frequency range of emitters is limited.

For these conditions, the predominant signal with the power $P_{max}=S_e\Pi_{max}$ at the receiver input belongs to the nearest emitter whose potential influence on the victim receiver has not been compensated by regulatory or technical measures. Since distances from the victim receiver to interferers are random, the magnitude P_{max} and the dynamic range D_P of oscillations in the ensemble of input signals are functionally related random values and can be characterized by the corresponding probability distributions. The type of these distributions is determined by models (2.2),(2.4).

3. Basic Probability Distribution of Signals Dynamic Range at the Receiver Input

For m-dimensional spatial distribution of emitters, the distance R_{min} to the nearest emitter may be defined as the maximum radius of the spherical receiver vicinity ΔU free from emitters Taking account of possible regulatory and

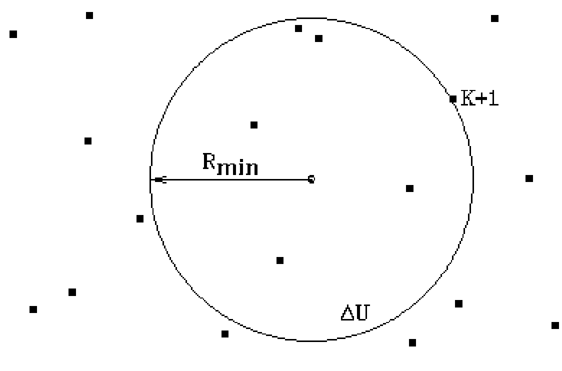

Fig. 1

technical ways to provide electromagnetic compatibility of neighboring radio stations (blanking, compensation, spatial EMI shielding, coordination of transmit times), which allow one to rule out the influence of signals from a certain number K of the nearest emitters (see Fig.1), the radius of this vicinity will be equal to the distance from the receiver to the $K+1$-th distant emitter. This emitter may be considered as the nearest interferer. As a result of this, the probability distribution function for ΔU becomes:

$$P(\Delta U) = 1 - \sum_{k=1}^{K} p_k(N_{\Delta U}) = p_{\geq(K+1)}, \quad (3.1)$$

where $N_{\Delta U}=\rho\cdot\Delta U$ is the average number of point emitters within the spherical vicinity ΔU, the probability $p_k(N_{\Delta U})$ is defined in (2.4), $p_{\geq(K+1)}$ is the probability that actual random sizes of the m-dimensional spherical area containing no less than $K+1$ point emitters will not exceed the size of ΔU (that is, the $K+1$ -th distant emitter will be located at the outer boundary of the spherical vicinity ΔU of the receiver location point). Thus the function (3.1) represents the probability that at least one more emitter beside K nearest emitters whose signals were compensated by regulatory or technical measures will be present within the considered receiver vicinity and this emitter will be the nearest emitter. If we differentiate (3.1), it is easy to see that for the accepted model (2.4) of random spatial distribution of emitters in the receiver vicinity, the probability distribution density for the volume (area) ΔU takes the form of the K-th order Erlang distribution:

$$w(\Delta U) = \frac{\rho^{K+1}\Delta U^{K}}{K!}\exp(-\rho\cdot\Delta U). \quad (3.2)$$

In general, $\Delta U = a_m R_{\min}^m$, $a_m = \pi^{m/2}\big/\Gamma(1+m/2)$ ($\Gamma(*)$ - gamma function, $m=1,2,3,...$). Therefore the probability distribution density for the distance R_{min} to the nearest interferer is:

$$w(R_{\min}) = \frac{\rho^{K+1}a_m^{K+1}m}{K!}R_{\min}^{(K+1)m-1}\exp(-\rho a_m R_{\min}^m),$$

$$R_{\min} \geq 0. \quad (3.3)$$

The mathematical form of this distribution allows for expansion of the definition domain of its parameters m, K to the whole domain of real positive numbers [**6,7**], which makes it possible to write its generalized differential and integral forms and the expression for moments of the arbitrary order n as:

$$w(R_{\min}) = \frac{G^H m}{\Gamma(H)}R_{\min}^{Hm-1}\exp(-GR_{\min}^m); \quad (3.4)$$

$$P(R_{\min}) = \frac{\gamma(H, GR_{\min}^m)}{\Gamma(H)}; \quad (3.5)$$

$$m_n(R_{\min}) = \frac{\Gamma(H+n/m)}{\Gamma(H)G^{n/m}}; \quad (3.6)$$

$$G = \rho a_m > 0, \quad H = K+1 > 0, \quad m > 0, \quad R_{\min} \geq 0;$$

$$\gamma(H, GR_{\min}^m) = \int_0^{GR_{\min}^m}\exp(-x)\cdot x^{H-1}dx$$ is the incomplete gamma function of the first kind.

$GR_{\min}^m = N_a(R_{\min})$ represents the average number of emitters within the m-dimensional spherical area (vicinity) ΔU with the radius R_{min} around the receiver location point.

Moreover, using the apparent functional relationship between the dynamic range D_P and the minimum distance $R_{\min}$

$$D_P = \left(c_v P_{etr}\right)\Big/\left(\Pi_{\min}R_{\min}^m\right),$$

we obtain from (3.4),(3.5) the law for probability distribution of the signals dynamic range at the receiver input; the distribution is determined by the level of the signal from the H-th distant emitter and this law has the form of the exponential-hyperbolic distribution [**6,7**]:

$$w(D_P) = \frac{N_a^H m}{v\Gamma(H)}D_P^{-(Hm+v)/v}\exp(-N_a D_P^{-m/v}); \quad (3.7)$$

$$P(D_P) = \frac{\Gamma(H, N_a D_P^{-m/v})}{\Gamma(H)}; \quad (3.8)$$

$$D_P \geq 0, N_a \geq 0, v > 0; m > 0;$$

$$\Gamma(H, N_a D_P^{-m/v}) = \int_{N_a D_P^{-m/v}}^{\infty}\exp(-x)\cdot x^{H-1}dx$$ is the incomplete gamma function of the second kind.

In this distribution, N_a represents the average number of emitters within the spherical area of potential interfering field interaction with the radius $R_{max}=(C_v P_{etr}/\Pi_{min})^{1/v}$ limited by the receiver sensitivity on the main receive channel in case the average spatial density of emitters within this whole area is constant and equal to the average density ρ of the random spatial distribution of emitters in the vicinity of the receiver:

$$N_a = GR_{\max}^m = \rho\pi^{m/2}R_{\max}^m\big/\Gamma(1+m/2) \geq 0. \quad (3.9)$$

The expression for moments of the distribution (3.7),(3.8) is:

$$m_n(D_P) = N_a^{nv/m}\Gamma(H - nv/m)\big/\Gamma(H), \quad (3.10)$$

$$H - nv/m > 0.$$

The expressions (3.7),(3.8),(3.10) thoroughly characterize probabilistic properties of the dynamic range of emissions (signals) at the receiver location point in the simplest case when EIRPs of emitters that are randomly located within the potential interfering interaction area are constant in time and equal in the direction to the radio receiver. In these expressions, $D_P \geq 0$, although the domain $D_P > 1$. The condition $D_P < 1$ means that if the Poisson model of spatial emitters location (2.4) is used then the probability that these emitters are absent within the area around the radio receiver with the radius R_{max} is not equal to zero for any finite ρ, R_{max} although this probability is generally extremely low.

It is necessary to point out that $n\nu/m \geq H$ and the distribution (3.7) have no initial moments in the most practically important cases. Nevertheless the obtained expressions make it possible to predict the dynamic range of signals during radio reception in space-scattered radio equipment groups on the basis of the evaluation of the upper boundary of the confidence interval $[0, D_{P0}]$ which includes the evaluated value of D_P with the probability p:

$$D_{P0} = \arg\{P(D_{P0}) = p\}, \qquad (3.11)$$

$$P(D_{P0}) = \Gamma(H, N_a D_{P0}^{-m/\nu}) / \Gamma(H).$$

For example, if in the considered situation the receiver antenna input dynamic range D_{in} is known then the probability $p(D_P > D_{in})$ that it will be exceeded by the dynamic range of input signals is determined by the simple relationship:

$$p(D_P > D_{in}) = 1 - \Gamma(H, N_a D_{in}^{-m/\nu}) / \Gamma(H). \qquad (3.12)$$

The distribution (3.7) is obtained using model (2.2) and the dynamic range of signals is determined using the power flux density (or power of individual signals at the receiver antenna input). If we use the propagation model of the type (2.2) based on the field intensity, in which the exponent is two times less than ν, then the form of expressions (3.7),(3.8),(3.10) and the meaning of their parameters will not change, but the condition for existence of moments in this distribution will be improved.

It is possible to consider the arguments in [**8**] based on fundamental properties of the model (2.4) that allow one to substantially improve limitations on applicability of models and relationships derived above; these limitations were assumed for derivation of the distribution (3.7),(3.8).

4. Signals Dynamic Range Probability Distribution as Sample Range Distribution

The signals dynamic range probability distributions derived above were based on the models (2.2),(2.4) that define the emitters spatial location randomness character and the statistical characteristics of strong signals ensemble at the reception point. If the formula of signals distribution by an energetic parameter may be defined for a certain spatial point then the dynamic range statistical characteristics for this signals ensemble can be directly determined on the basis of solving the known problem of sample range estimation [**9**] for random values of the power flux density (or field strength or signal level at the receiving antenna output) of the signals distributed in accordance with this formula.

In particular, if the random spatial emitters distribution model (2.4) is valid and the average random spatial emitter distribution density is constant $(\rho = const)$ it is observed that the probability distribution density of signals over the power flux density at a certain arbitrarily selected spatial location point (the receiver location point) has the form of the hyperbolical distribution [**10**]:

$$w(\Pi) = \frac{m \Pi_{\min}^{m/\nu}}{\nu \Pi^{1+m/\nu}}, \qquad \Pi \geq \Pi_{\min}, \qquad (4.1)$$

where Π_{min} is the receiver sensitivity limit. If the ensemble of values of a random variable Π distributed according to (4.1) is represented as the variational series then it is possible to use known methods to determine the probability distribution for the N-th order statistics Π_N of these series as well as to determine the probability distribution function $F_N(D)$ given that the number of signals is equal to N:

$$F_N(D) = \left(1 - D^{-m/\nu}\right)^N, \; D = \Pi_N / \Pi_{\min} \geq 1. \qquad (4.2)$$

The comparative analysis of prediction results for the probable signals dynamic range (obtained for the confidence probability $p \geq 0.9$ using models (3.8) and (4.2) ($N_a = N$, $H = 1$) shows that the coincidence of estimations obtained using these models is satisfactory, especially for high (large) values of N. In particular, for $F_N(D) = 0.9$, $N = 1000$ the results coincidence accuracy reaches ***0.1%.***

However, it is important to point out the different meaning of N_a in (3.9), where this variable is a real number (the average number of signals at the receiver input) and N in (4.2) where this variable is an actual number of signals at the receiver input and must be considered as a random integer number with regard to the expected radio environment. Therefore it is necessary to take account of the probability that the specific actual number of signals will be present at the receiver input whose levels exceed the threshold level Π_{min} in order to use the model (4.2) for estimation of the probable dynamic range of signals whose emitters are located within the potential interfering interaction area. Taking into account the assumed probabilistic character (2.4) of signal ensemble formation, we obtain the following expression for the signals dynamic range probability distribution function:

$$F(D) = \sum_{N=0}^{\infty} p_N(N_a) F_N(D) = \\ = \exp(-N_a) \sum_{N=0}^{\infty} \frac{N_a^N}{N!} \left(1 - D^{-m/\nu}\right)^N, \qquad (4.3)$$

where N_a is the parameter N_a from the distribution (3.7),(3.8), defined by one of the above-described methods.

The model (4.3) may be used for making estimations of the type $F(D)$ for any signal probability distribution over the energetical parameter. This model can also use the representation $F_N(D) = F^N(\Pi/\Pi_{min})$ where $F(\Pi/\Pi_{min})$ is the probability distribution function for the ratio of the signal power flux density Π to the receiver sensitivity limit Π_{min}. If the signals sample range N is determined and there is no sense in using the model (2.4) the relationship (4.3) degenerates into the equality $F(D)=F_N(D)$.

In the model (3.7),(3.8) the parameter H characterizes the possibility for reducing (in the sense of probabilistic and statistical characteristics) the input signals dynamic range due to suppression of the $H-1$ strongest signals at the reception point. Let us show that this factor may be taken into account when the sampling method is used for prediction of the signals dynamic range at the reception point.

If N signals distributed over the energetical parameter in accordance with (4.1) are present at the receiver input then conversion of the sample range $\Pi_1,\Pi_2,...,\Pi_N$ into the variational series $D_{(1)}=\Pi_{(1)}/\Pi_{min}$, $D_{(2)}=\Pi_{(2)}/\Pi_{min}$,..., $D_{(N)}=\Pi_{(N)}/\Pi_{min}$; $\Pi_{(1)}<\Pi_{(2)}<...<\Pi_{(N)}$ using the known rules makes it possible to obtain the probability distribution density of the k-th order statistics $D_{(k)}$ of these series:

$$w\left(D_{(k)}\right)=\frac{m\Gamma(N+1)}{\nu\Gamma(k)\Gamma(N-k+1)}\left[1-D_{(k)}^{-m/\nu}\right]^{k-1} * D_{(k)}^{-\frac{m}{\nu}(N-k+1)-1} = \frac{m\Gamma(N+1)}{\nu\Gamma(H)\Gamma(N-H+1)}\left[1-D_{(k)}^{-m/\nu}\right]^{N-H} D_{(k)}^{-\frac{m}{\nu}H-1}, \quad k=N-H+1. \qquad (4.4)$$

Let us compare (4.4) and (3.7). We substitute $D=D_{(k)}$ into the model (3.7), denote this model as $w'(D_{(k)})$ and denote the sampling model (4.4) as $w''(D_{(k)})$ and then obtain their relationship:

$$S=\frac{w''\left(D_{(k)}\right)}{w'\left(D_{(k)}\right)}=\frac{\Gamma(N+1)\left[1-D_{(k)}^{-m/\nu}\right]^{N-H}}{\Gamma(N-H+1)\exp\left(-ND_{(k)}^{-m/\nu}\right)N^H}.$$

The most interesting case is the case of high (large) values of N, $D_{(k)}$. If $D_{(k)}>>N$ then we have:

$$\left[1-D_{(k)}^{-m/\nu}\right]^{N-H}\approx 1-(N-H)D_{(k)}^{-m/\nu}\to 1;$$

$$\exp\left(-ND_{(k)}^{-m/\nu}\right)\approx 1-ND_{(k)}^{-m/\nu}\to 1;$$

$$\lim_{N/H\to\infty}[N!]\Big/\left[(N-H)!N^H\right]\to 1.$$

Thus for $H<<N<<D_{(k)}^{m/\nu}=\Re$ we have $\lim_{\Re\to\infty} S\to 1$, that is, coincidence of the models (3.7) and (4.4) is observed. This fact allows us to use (4.4) in order to predict the probable signals dynamic range for confidence probability levels of $p\geq 0.9$ and also in those cases where the system level estimation of signals dynamic range is based on spatial emitters location modeling using the model (2.4) and we have to take into account the interference amount randomness (random number of interference signals) at the receiver input.

The integrated form of the distribution (4.4) and the expression for its (initial) moments of the n-th order are:

$$F_N\left(D_{(k)}\right)=1-\frac{B'_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1)}{B(H,N-H+1)}=\frac{B''_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1)}{B(H,N-H+1)}=1-I_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1); \qquad (4.5)$$

$$m_n\left(D_{(k)}\right)=\frac{\Gamma(N+1)\Gamma(H-\nu n/m)}{\Gamma(H)\Gamma(N-\nu N/m+1)}=\prod_{i=1}^{\nu n/m}\frac{N+1-i}{H-i},\quad H>\nu n/m; \qquad (4.6)$$

in these relationships, $B(H, N-H+1)$ is the beta function (Eulerian integral of the first kind); $B'_{\left(D_{(k)}^{-m/\nu}\right)}$, $B''_{\left(D_{(k)}^{-m/\nu}\right)}$, $I_{\left(D_{(k)}^{-m/\nu}\right)}$ are incomplete beta functions in various variants:

$$\left.\begin{aligned}
&B(H,N-H+1)=\int_0^1 t^{H-1}(1-t)^{N-H}dt=\frac{\Gamma(H)\Gamma(N-H+1)}{\Gamma(N+1)};\\
&B'_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1)=\int_0^{D_{(k)}^{-m/\nu}} t^{H-1}(1-t)^{N-H}dt;\\
&B''_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1)=\int_{D_{(k)}^{-m/\nu}}^1 t^{H-1}(1-t)^{N-H}dt;\\
&I_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1)=\frac{B'_{\left(D_{(k)}^{-m/\nu}\right)}(H,N-H+1)}{B(H,N-H+1)}.
\end{aligned}\right\} \qquad (4.7)$$

It is apparent that when investigating the statistical characteristics of the dynamic range as the sample range of values distributed according to (4.1) the best fit to (3.7),(3.8) can be obtained by weighing the hyphotheses (4.4),(4.5) with the probabilities (2.4). In this case the signals dynamic range probability distribution function for known N_a, H is:

$$F_{N_a}\left(D_{(k)}\right)=\sum_{N=0}^{H-1}p_N(N_a)+\sum_{N=H}^{\infty}p_N(N_a)F_N\left(D_{(k)}\right),\quad k=N-H+1. \qquad (4.8)$$

By and large, the following conclusions may be drawn comparing the models (3.7)-(3.10) and the models (4.4)-(4.8):

a) There is no noticeable advantage gained by using the model (4.4)-(4.8), as compared to the model (3.7)-(3.10), for signals dynamic range prediction when spatial emitters location parameters and propagation conditions are given. This is true because (initial) moments of (3.10) and (4.6) exist under the same condition $(H>\nu n/m)$, and for low (small) values of H the prediction technique for both models assumes that the upper limit of the signals dynamic range values confidence interval is estimated for the given confidence probability in accordance with (3.11) and the probability that the receiver dynamic range is exceeded by the input signals dynamic range is estimated on the basis of (3.12);

b) It is preferable to use the model (4.4)-(4.8) when the sample range of signal level values distributed according to (4.1) is fixed. In these cases the computational difficulties related to estimation of the incomplete beta functions and the full beta function in (4.7) may be overcome using known asymptotic representations, for instance, on the basis of computing the corresponding values of the hypergeometric Gaussian function;

c) Prediction of signals dynamic range statistical characteristics based on the probability distribution density analysis of the k-th order statistics $D_{(k)}$ of the variational series $D_{(1)}=\Pi_{(1)}/\Pi_{min}$, $D_{(2)}=\Pi_{(2)}/\Pi_{min}$,..., $D_{(N)}=\Pi_{(N)}/\Pi_{min}$, using the technique described above is preferable when the form of probability distribution over the signals energetical parameter at the reception point and the number of signals are determined from the obtained radio monitoring data and other information (concerning characteristics of spatial emitters distribution and propagation conditions) is unavailable;

d) Signals dynamic range analysis at various points of receiver structure (at the receiving antenna input and output, at the preselector output etc) taking into account the possibility to suppress a certain number K of the strongest signals $(H=K+1>1)$ assumes that the models and relationships presented above are used consequently as follows:

- estimation of electromagnetic fields dynamic range at the receiver location point using the model (3.7)-(3.10) or the model (4.4)-(4.8). The model choice depends on the available initial information in accordance with a) and b);
- estimation of the expected average signals number $(N_a)_i$ $i=1,2,...$ at the analyzed receiver internal points which takes into account the characteristics of receiver input elements linear selectivity by non-energetical parameters [**6,8**];
- estimation of signals dynamic range at the analyzed points of receiver structure which uses (3.11),(3.12) with the model (3.7)-(3.10) or (4.4)-(4.8) and is based on the prediction results for the average number $(N_a)_i$ of signals at these points.

5. Mathematical Interpretation of Results

The basic mathematical model that facilitated the results presented above is related to investigation and utilization of laws concerning mutual arrangement and interaction of point elements in m-dimensional Euclidean space given the Poisson probabilistic character of random uniform spatial distribution of these elements (model (2.4)) and given that the field of spatial elements interaction is described by the relationship of the type (2.2). The generalizing character of this model facilitates uniform interpretation of notation forms and generation mechanisms for a significant number of known probability distributions widely used in statistics.

It is true that the models (3.5), (3.8) and (4.5) derived above are the three-parameter models and hence are generalizing models with regard to a large group of known one- and two-parameter distribution functions. These models are generated using Eulerian integrals of the first and second kind (full and incomplete gamma and beta functions). The generalization of mathematical constructions of these models given below forms a group of three-parameter models whose utilization for problems related to approximation, analysis and synthesis of random variable probability distributions is of a certain interest. In addition, unlike the known Pearson and Johnson curves the discussed models are provided with the uniform interpretation scheme and the mechanism for their generation based on random uniform m-dimensional arrangement and interaction of point objects.

Let us write the distribution function of a random variable x as:

$$P_1(x)=\frac{\int_a^{f(x)}\Phi(z)dz}{\int_a^b\Phi(z)dz},\quad f'_x(x)=\frac{df(x)}{dx}>0\,; \tag{5.1}$$

$$P_2(x)=\frac{\int_{f(x)}^{b}\Phi(z)dz}{\int_a^b\Phi(z)dz},\quad f'_x(x)<0\,, \tag{5.2}$$

where $\Phi(z)$ is a certain function which does not change its sign and can be integrated on the interval $[a,b]$; this interval corresponds to the domain of possible values of the continuous monotonous function *f(x)*. This distribution is characterized by density

$$w(x)=\left[\operatorname{sgn}\{f_x^{'}(x)\}\cdot \Phi[f(x)]\cdot f_x^{'}(x)\right]\Big/\int_a^b \Phi(z)dz \qquad (5.3)$$

Using Eulerian integrals of the first and second kind in the numerator and denominator positions of the relationships (5.1),(5.2), for the power function of the type *f(x)* and $x\geq 0$ we obtain the following variants of the distribution (5.1)-(5.3):

$$P_1(x)=\frac{\gamma(H,Gx^m)}{\Gamma(H)};\quad w_1(x)=\frac{G^H m}{\Gamma(H)}x^{Hm-1}\exp\left(-Gx^m\right); \qquad (5.4)$$

$$P_2(x)=\frac{\Gamma(H,Sx^{-n})}{\Gamma(H)};\quad w_2(x)=\frac{S^H n}{\Gamma(H)}x^{-Hn-1}\exp\left(-Sx^{-n}\right); \qquad (5.5)$$

$$P_3(x)=\frac{B'_{x^r}(H,W)}{B(H,W)};\quad w_3(x)=\frac{rx^{rH-1}(1-x^r)^{W-1}}{B(H,W)}; \qquad (5.6)$$

$$P_4(x)=\frac{B''_{x^{-c}}(H,W)}{B(H,W)}=1-I_{x^{-c}}(H,W);\quad w_4(x)=\left[cx^{-cH-1}(1-x^{-c})^{W-1}\right]\Big/\left[B(H,W)\right] \qquad (5.7)$$

Each of these distributions is briefly characterized below.

1. The distribution (5.4) coincides with the distribution (3.4)/(3.5); this distribution is characterized by the parameters ***m, G, H*** and represents the probability distribution for the distance ***x*** from an arbitrary point selected as the origin of coordinates in ***m***-dimensional Euclidean space to ***H***-th distant from the origin of coordinates point element given the Poisson probabilistic character of random uniform spatial distribution of these elements with the average spatial density $\rho=G/a_m=G\Gamma(1+m/2)/\pi^{m/2}$. The following known distributions can be cited as special cases of this model:

- ◊ for ***m=1*** - the gamma distribution and its special cases : the χ^2 distribution ***(G=0.5),*** the exponential power series distribution ***(G=1), (H=1),*** the Erlang exponential distribution (at integer values of ***H***);
- ◊ for ***m=2*** - the Nakagami distribution and the multidimensional vector module distribution, as well as their special cases: the χ distribution ***(G=0.5),*** the Maxwell distribution ***(H=1.5),*** the Rayleigh distribution ***(H=1)*** and the Gauss distribution ***(H=0.5)***;
- ◊ for ***H=1*** - the Weibull distribution.

The distribution (5.4) is derived by substituting of the Eulerian integral of the second kind (gamma function) into the numerator and denominator of (5.1) for $f(x)=Gx^m\geq 0, f_x^{'}=Gmx^{m-1}>0; H\neq 0,\ \ ,-1,-2,...$.

2. The distribution (5.5) coincides with the distribution (3.7)/(3.8); this distribution is characterized by the parameters ***n,S,H*** and represents the probability distribution for the energetical parameter ***x*** of the field created by the ***H***-th distant point element at an arbitrary point selected as the origin of coordinates in ***m***-dimensional Euclidean space if

- ◊ the random uniform spatial distribution of these elements is of Poisson probabilistic character;
- ◊ the considered field parameter $x=\Pi_H/\Pi_{min}$ is inversely proportional to power ***m/n*** of the distance R_H to the ***H***-th distant point element: $\Pi_H=const/R_H^{m/n}$;
- ◊ the parameter ***S*** represents the average number of elements in the area with the radius R_{max}, such that if a certain element is located within this area then the parameter Π_H of its field in the center of this area can be measured by the equipment with the sensitivity limit Π_{min} .

The distribution (5.5) is derived by substituting of the Eulerian integral of the second kind (gamma function) into the numerator and denominator of (5.2) for $f(x)=Sx^{-n}\geq 0, f_x^{'}=-Snx^{-n-1}<0; H\neq 0,\ \ ,-1,-2,...$. In particular, at ***n=1, S>0, H>0*** it degenerates into the exponential hyperbolic distribution from the Pearson (curve) family (curve type V according to the conventional classification).

3. The distribution (5.6) is derived by substituting of the Eulerian integral of the first kind (beta function) into the numerator and denominator of (5.1) for $f(x)=x^r\in[0,1]; f_x^{'}=rx^{r-1}\geq 0; H>0, W>0.$ At ***r=1,*** it takes the form of the beta distribution (curve type I of Pearson curves) and can be interpreted as the distribution of the ***W***-th order statistics of the variational series $x_{(1)},x_{(2)},...,x_{(W)},...,x_{(N)}$, $W=N-H+1,$ where the series terms $x_{(i)}=R_{(i)}/R_{max}$ represent the relative distance from the observation point (origin of coordinates) to each of ***N*** point elements in ***m***-dimensional Euclidean space that are really within the spherical area with the radius R_{max} (if the random uniform spatial distribution of these elements is of Poisson probabilistic character).

4. The distribution (5.7) is derived by substituting of the Eulerian integral of the first kind (beta function) into the numerator and denominator of (5.1) for $f(x)=x^{-c}\in[0,1]; f_x^{'}=-cx^{-c-1}<0; H>0, W>0.$ It coincides with the distribution (4.4)/(4.5); this distribution is characterized by the parameters ***c,H,W*** and represents the distribution of the ***W***-th order statistics of the variational series $x_{(1)},x_{(2)},...,x_{(W)},...,x_{(N)}$, $W=N-H+1,$ where the series terms $x_{(i)}=\Pi_{(i)}/\Pi_{min}$ represent the values of the relative energetical parameter of the field created by the ***H***-th distant point element at an arbitrary point of ***m***-dimensional Euclidean space selected as the origin of coordinates if

- ◊ the random uniform spatial distribution of these elements is of Poisson probabilistic character and there are N point elements that are really within the spherical area with the radius R_{max} and the field parameter $\Pi_{(i)}$ of each of these elements can be measured by the equipment with the sensitivity limit Π_{min};

◊ the considered field parameter $x_{(H)}=\Pi_{(H)}/\Pi_{min}$ is inversely proportional to power m/c of the distance $R_{(H)}$ to the H-th distant point element: $\Pi_{(i)}=const/R_{(H)}^{m/c}$.

At $W=1$, the distributions (5.7),(5.8) degenerate into the power series distribution which, for instance, may take the form of the hyperbolical distribution (4.1). It can also be easily observed that models (5.7),(5.8) are related to the generalized arcsine distribution, the Snedecor distribution, the Pareto distribution, the Student distribution, the Cauchy distribution.

6. Practical applications

The practicability of the models is illustrated by the results of this technique utilization for prediction of reception conditions in FDMA, TDMA and CDMA networks presented in [**6,7**]. In particular,

a) for the FDMA network the following estimated values of dynamic signals range at the reception point were obtained (the probability that these estimated values shall not be exceeded is $p=0.9$ and $p=0.99$ correspondingly): $D(0.9)=20lg(1.88N_a)$[dB], $D(0.99)= =lg(6.73N_a)$[dB]; the difference between these estimated values for the same value of N_a is approximately 11dB;

b) for reception on the TDMA network frequency channels, the upper limit for the dynamic range D of signals in the main receive path (with the probability p that this limit will not be exceeded) can be determined using the expression $D\sim20lg(-N/ln\ p)$ [dB]. In particular, the estimated values of D for urban area $(\nu=4)$ channels with $N=4$ (TETRA), $N=8$ (GSM) and $N=12$ (DECT) are 32-42dB for $p=0.9$ and 52-62 dB for $p=0.99$;

c) the utilized models make it possible to relate cell size variations in the CDMA network to density variations of operating mobile stations and statistical characteristics of dynamic range of their signals [**7**].

Utilization of the models (5.4)-(5.7) for experimental data approximation makes it possible (in some cases) not only to choose the analytical form which provides the best approximation of the random variable histogram but also to presume its possible generation mechanism. The presented models make it possible to implement space-scattered networks simulation deployed on transportation facilities $(m=1)$, over a certain area $(m=2)$, or within a multistoried building $(m=3)$ as well as to enhance the application areas of the probabilistic models derived above for their application to solving various computer simulation and statistical data analysis problems.

In particular, the discussed models may be of substantial interest with regard to implementation of a technique and software for analyzing cellular networks with the use of the Monte-Carlo method [**11**] and for investigating how the spatial location randomness of the network components influences the communication quality, intrasystem electromagnetic compatibility (EMC) and spectrum usage efficiency.

The presented approaches make it possible to use various scenarios in the dynamic range prediction of signals that create the radio environment at the reception point. Author tends to think that the materials presented above have the potential for further development and provide opportunities for investigation of EMC problems in mobile communication systems.

7. References

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